sure $p = 10 \times 10^{-5}$ dyne/cm², the resulting deflection is negligible in the cases already treated; for very slender rods, solar pressure effect may predominate because of dependence on I^{-1}.

Concluding Remarks

The discussion in this note allows one to draw conclusions on the plausibility of assumptions made by earlier investigators. The numerical results show that thermal deflections are relatively insensitive to the form of variation of the heat input function in the axial direction.

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Torsional Dynamics of an Axially Symmetric, Two-Body, Flexibly Connected Rotating Space Station

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Nomenclature

A, C	= total moments of inertia of parts 1 and 2 about any axis in x, y plane, and about z axis, respectively
C_1, C_2	= moments of inertia of parts 1 and 2 about z axis
E	= constant of integration (amplitude of γ)
f	= viscous damping coefficient, torque/angular velocity
g	= torque component applied to part 1 by part 2 along z direction
ϑ_1, ϑ_2	= inertia dyadics of parts 1 and 2 referred to x, y, z axes
$\mathbf{i}, \mathbf{j}, \mathbf{k}$	= unit vectors along the x, y, z body axes, respectively
$\mathbf{J}$	= applied angular impulse vector [see Eq. (19)]
K	= torsional spring constant, torque/angular displacement
$\mathbf{L}$	= total system angular momentum vector
$\mathbf{N}$	= applied torque vector to system
$\mathbf{n}_{xy}$	= unit vector in direction of component of $\boldsymbol{\omega}_1$ on x, y plane
p	= frequency of oscillation of γ
q	= exponential decay constant
t	= time
X, Y, Z	= axes of the inertial coordinate system; Z axis coincides with angular momentum vector
x, y, z	= principal axes of entire system, fixed in part 1 with origin at system center of mass; z axis is axis of revolution
γ	= angular rotation between parts 1 and 2
ζ	= constant of integration (phase angle associated with γ)
θ, φ, ψ	= Euler angles in zZ, xy, and XY planes, respectively (Fig. 1); θ is precession half-cone angle
λ	$\equiv \Omega_z(C - A)/A$
ω	$\equiv \omega_x + i\omega_y$
$\boldsymbol{\omega}$	= angular velocity vector
Ω, Ω_z	= constants of integration; Ω is complex

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Subscripts and superscript

x, y, z	= components along x, y, z axes; xy denotes component in xy plane
1, 2	= portion associated with or applied to part 1 or 2, respectively
0	= value immediately prior to applied impulse

Introduction

As a result of the large size required for possible rotating manned space stations, as well as weight limitations on structural elements, these vehicles may be quite flexible. In this note the dynamic effects of concentric torsional oscillations of a vehicle (Fig. 1) are discussed. Constraints are considered to prevent relative motion of the two interconnected rigid bodies in any direction other than in concentric rotation about the axis of revolution. Thomson and Reiter[1] have shown that the general motion of a free elastic body of revolution results in a change in the precession cone angle. However, for the configuration considered herein, gyroscopic effects do not induce elastic vibration, and it is shown that the over-all precessional motion is independent of the relative oscillations of the two members.

Analysis

The center of mass of parts 1 and 2 (Fig. 1) need not coincide in the z direction. The axes are selected as fixed in part 1 or its imaginary extension (if the system center of mass does not fall within the confines of that part). The angular velocities are

$$\boldsymbol{\omega}_1 = \mathbf{i}\omega_x + \mathbf{j}\omega_y + \mathbf{k}\omega_z \qquad \boldsymbol{\omega}_2 = \boldsymbol{\omega}_1 + \mathbf{k}\dot{\gamma} \tag{1}$$

By summing the angular momentum of all of the particles and noting that the moments of inertia of part 2 are constants in the body reference frame, the system angular momentum is

$$\mathbf{L} = \vartheta_1 \cdot \boldsymbol{\omega}_1 + \vartheta_2 \cdot \boldsymbol{\omega}_2 = \mathbf{i}A\omega_x + \mathbf{j}A\omega_y + \mathbf{k}(C\omega_z + C_2\dot{\gamma}) \tag{2}$$

The components of the torque along the body reference axes are obtained by projecting the spatial derivative of $\mathbf{L}$ on this coordinate system:

$$\mathbf{N} = (d\mathbf{L}/dt) + \boldsymbol{\omega}_1 \times \mathbf{L} \tag{3}$$

For the case of the free system, $\mathbf{N}$ is zero. In addition, g is assumed to be a function of γ and $\dot{\gamma}$ only:

$$A\dot{\omega}_x + (C - A)\omega_y\omega_z + C_2\dot{\gamma}\omega_y = 0 \tag{4}$$

$$A\dot{\omega}_y - (C - A)\omega_x\omega_z - C_2\dot{\gamma}\omega_x = 0 \tag{5}$$

$$C\dot{\omega}_z + C_2\ddot{\gamma} = 0 \tag{6}$$

$$C_1\dot{\omega}_z = g(\gamma, \dot{\gamma}) \tag{7}$$

From Eqs. (6) and (7)

$$(C_1C_2/C)\ddot{\gamma} + g(\gamma, \dot{\gamma}) = 0 \tag{8}$$

$$\omega_z = \Omega_z - (\dot{\gamma}C_2/C) \tag{9}$$

where Ω_z is a constant of integration. With the use of this result, the solution to Eqs. (4) and (5) is

$$\omega/\Omega = \exp\{\textstyle\int i[\lambda + (\dot{\gamma}C_2/C)]dt\} \tag{10}$$

where $\omega \equiv \omega_x + i\omega_y$, $\lambda \equiv \Omega_z(C - A)/A$, and Ω is a complex constant of integration. Since the modulus of the right-hand side of Eq. (10) is unity, the projection of $\boldsymbol{\omega}_1$ and $\boldsymbol{\omega}_2$ on the x, y plane is a constant:

$$|\omega| = |\Omega| = (\omega_x{}^2 + \omega_y{}^2)^{1/2} \equiv \Omega_{xy} \tag{11}$$

From Eqs. (2) and (9)

$$\mathbf{L} = A(\mathbf{i}\omega_x + \mathbf{j}\omega_y) + \mathbf{k}C\Omega_z = A\,\mathbf{n}_{xy}\Omega_{xy} + \mathbf{k}C\Omega_z \tag{12}$$

It is seen that the projection L_{xy} of $\mathbf{L}$ on the x,y plane is

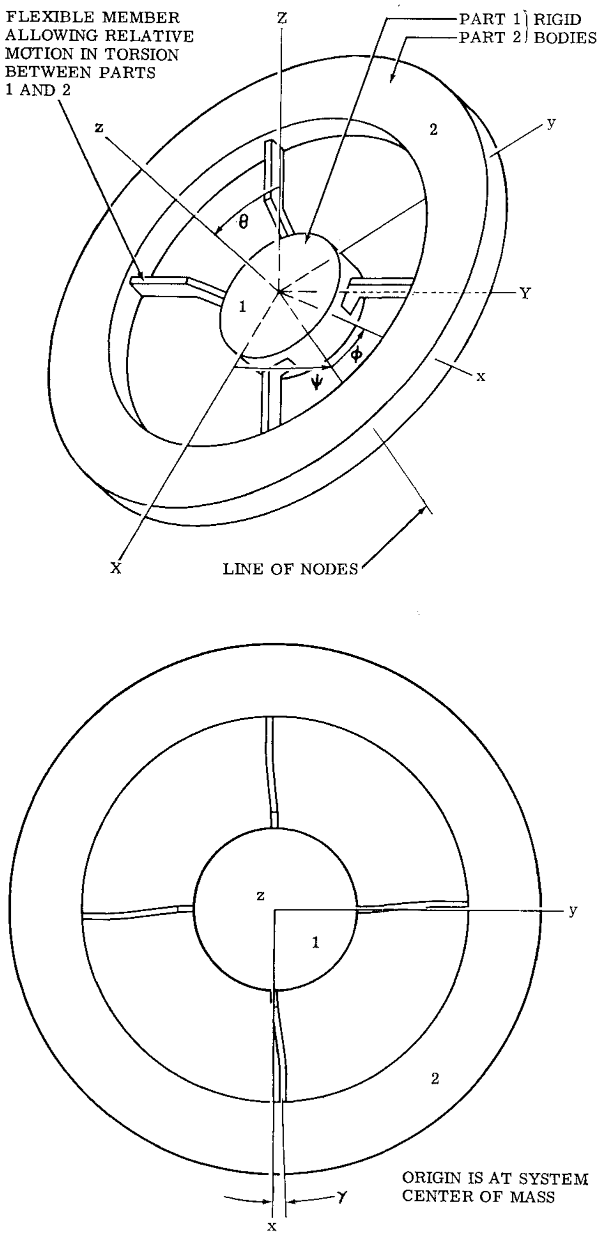

Fig. 1 System model and coordinates.

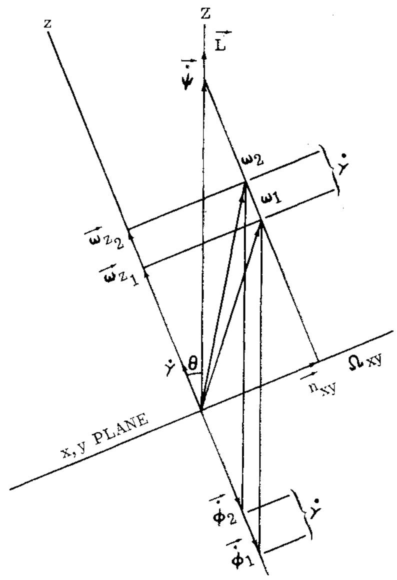

Fig. 2 System angular velocity diagram.

colinear with the projections of $\boldsymbol{\omega}_1$ and $\boldsymbol{\omega}_2$. In consequence, $\mathbf{L}$, $\boldsymbol{\omega}_1$, $\boldsymbol{\omega}_2$, and the z axis lie in the same plane. Let the Z inertial axis be oriented along, and directed toward, $\mathbf{L}$. From Eq. (12), the half-cone angle between the Z and z axes is

$$\tan\theta = L_{xy}/L_z = A\Omega_{xy}/C\Omega_z \qquad (13)$$

The Euler angles used in locating part 1 are shown in Fig. 1. Since θ is constant,

$$\omega_x = \dot{\psi}\sin\theta\sin\varphi \qquad \omega_y = \dot{\psi}\sin\theta\cos\varphi$$

$$\omega_z = \dot{\varphi} + \dot{\psi}\cos\theta \qquad (14)$$

where $\dot{\varphi}$ and $\dot{\psi}$ are the angular velocity components along z and Z. From Fig. 2 [or alternately by substituting Eqs. (14) into (11)], using Eq. (12) to obtain $\sin\theta$,

$$\dot{\psi} = \Omega_{xy}/\sin\theta = L/A \qquad (15)$$

Both the precessional velocity ($\dot{\psi}$) and θ are constants and are given by the same relations [Eqs. (15) and (13)] as in the rigid-body theory. Solving the last equation of (14) for $\dot{\varphi}$ and then using Eqs. (9, 15, and 13), the spin velocity of the axes becomes

$$\dot{\varphi} = -\lambda - \dot{\gamma}C_2/C \qquad (16)$$

from which $\dot{\varphi}_2$ may be obtained simply by adding $\dot{\gamma}$ to this value.

Assuming that the solutions to Eq. (8) are stable, so that the phase trajectories decay with time, the following conclusions may be ascertained. The free motion of the system may be described by a constant θ, a constant $\dot{\psi}$, and varying spin velocities of the two members that approach the same constant $-\lambda$ as the time varying portions decay. The solutions indicate that, unless γ or $\dot{\gamma}$ is excited initially, the system will behave as a rigid body of revolution. Even if these values are excited initially, the results approach those for a rigid body as time increases.

As an example, suppose the relative motion between parts is governed by a linear spring and damper. Then

$$g(\gamma, \dot{\gamma}) = f\dot{\gamma} + K\gamma \qquad (17)$$

The solution to Eq. (8) is

$$\gamma = E\exp(-qt)\cos(pt - \zeta) \qquad (18)$$

where $q \equiv f(C_1^{-1} + C_2^{-1})/2$, $p \equiv (2Kq/f - q^2)^{1/2}$, and E and ζ are constants of integration. This solution may be substituted readily into Eqs. (10, 9, and 16) to obtain ω_x, ω_y, ω_z, and $\dot{\varphi}$ as functions of time.

Response to impulse torques

The following angular impulse is considered:

$$\mathbf{J} = \lim_{t_2 \to t_1} \int_{t_1}^{t_2} \mathbf{N}dt \qquad (19)$$

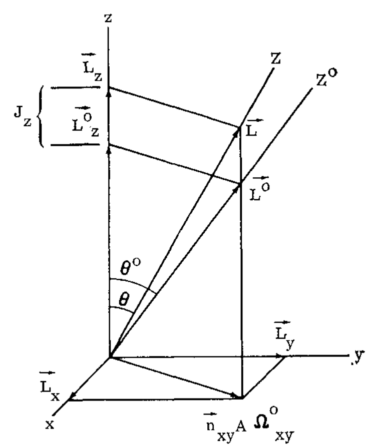

Fig. 3 Angular momentum change due to J_z applied to part 2.

$\mathbf{N} = (d\mathbf{L}/dt) + \omega_1 \times \mathbf{L}$, and in order that $\mathbf{J}$ remain finite, angular accelerations, but not angular velocities, are permitted to tend toward infinity during the limiting process; thus $\mathbf{J} = \Delta\mathbf{L}$ and

$$J_x = A\Delta\omega_x \qquad J_y = A\Delta\omega_y \qquad J_z = C\Delta\omega_z + C_2\Delta\dot{\gamma}$$
$$J_{z_1} = C_1\Delta\omega_z \qquad J_{z_2} = J_z - J_{z_1} \tag{20}$$

Hence, given any $\mathbf{J}$, new ω's may be computed which may be used as initial conditions in the solutions previously obtained for the free motion of the system. It is seen that, if γ and $\dot{\gamma}$ are initially zero, and if $J_{z_1} = J_{z_2} = 0$, then γ is not excited, and the system will respond as if it were a rigid body.

As an example, J_{z_2} is considered with γ and $\dot{\gamma}$ initially zero, a situation that might accompany a correction made by the spin control jets of a rotating space station. From the first, second, and fourth expressions of Eqs. (20), ω_x, ω_y, and ω_z remain initially unchanged by the impulse, since the associated impulse components are zero; $\dot{\gamma}$ changes in accordance with the remaining relations:

$$J_z = C_2\Delta\dot{\gamma} = C_2\dot{\gamma} \qquad \text{at} \quad t = 0^{(+)} \tag{21}$$

The angular velocity components, prior to application of the impulse, are defined as $\Omega_{xy}{}^0$ and $\Omega_z{}^0$. From Eqs. (2) and (21), the angular momentum following the impulse is

$$\mathbf{L} = \mathbf{n}_{xy}A\Omega_{xy}{}^0 + \mathbf{k}(C\Omega_z{}^0 + J_z) \tag{22}$$

The system immediately begins precessing about the new angular momentum vector (Fig. 3); θ may be obtained from

$$\tan\theta = A\Omega_{xy}{}^0/(C\Omega_z{}^0 + J_z) \tag{23}$$

Since $\tan\theta$, prior to the impulse, is $(A\Omega_{xy}{}^0)/(C\Omega_z{}^0)$, the angular variation is given by

$$\tan(\theta^0 - \theta) = A\Omega_{xy}{}^0 J_z/[(C\Omega_z{}^0)^2 + C\Omega_z{}^0 J_z + (A\Omega_{xy}{}^0)^2] \tag{24}$$

The precessional velocity is changed in accordance with Eq. (15)

$$\dot{\psi} = L/A = [(A\Omega_{xy}{}^0)^2 + (C\Omega_z{}^0 + J_z)^2]^{1/2}/A \tag{25}$$

In view of the values of γ and $\dot{\gamma}$ immediately after application of the impulse, Eq. (9) reveals that $\Omega_z = \Omega_z{}^0 + J_z/C$; thus, from Eq. (16),

$$\dot{\varphi} = -(\Omega_z{}^0 + J_z/C)(C/A - 1) - \dot{\gamma}C_2/C \tag{26}$$

It may be verified that the new θ and $\dot{\psi}$ are the same values that would occur if the system were rigid and subjected to the same angular impulse $\mathbf{k}J_z$ under the same initial conditions. Also, again assuming that the solutions to Eq. (8) are stable, $\dot{\varphi}$ and $\dot{\varphi}_2$ approach the corresponding values for the rigid system as time increases.

If $g(\gamma, \dot{\gamma})$ is linear, as in the previous example, then, from Eq. (18), $\zeta = \pi/2$, and $E = J_z/C_2p$. Hence, the relative angular motion and the spin velocities of parts 1 and 2 are, respectively,

$$\gamma = (J_z/C_2p)\exp(-qt)\sin pt \tag{27}$$

$$\dot{\varphi} = -[\Omega_z{}^0 + (J_z/C)][(C - A)/A] - (J_z/Cp)\exp(-qt)(q\sin pt - p\cos pt) \tag{28}$$

$$\dot{\varphi}_2 = -[\Omega_z{}^0 + (J_z/C)][(C - A)/A] - [(1/C_2) - (1/C)](J_z/p)\exp(-qt)(q\sin pt - p\cos pt) \tag{29}$$

Conclusions

The free motion is characterized by a relative elastic vibration between members which decays if an energy dissipation mechanism is present, whereas the over-all system precesses at a uniform rate about the system angular momentum vector and at a constant half-cone angle θ. When there is no initial vibration, an applied impulse torque, which lies in the x, y plane of the body axes, does not induce elastic motion, and the system responds as if it were rigid. If the impulse torque, applied to the outer portion, contains a z component, the new θ and precessional velocity are the same values as would occur for a rigid body, whereas the spin velocity approaches the corresponding rigid-body value as time increases.

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Handling Qualities Criteria for Manned Spacecraft Attitude-Control Systems

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In specifications of system performance and handling qualities criteria for spacecraft, there does not yet appear to be common definitions of maneuvers, forcing functions, or even the control systems. The precise definitions of control modes and maneuvers are usually given in system and mission specific terms that do not permit comparisons among different systems and missions. This occurs even with an organization. When the differences between local definitions are superimposed, it is nearly impossible to compare or utilize data from various sources. For example, the direct control of reaction jets by an astronaut has been referred to as direct manual, reaction-jet, acceleration, on-off, and manual pulse-width modulation.

In the first phase of this program,[1] an attempt was made to define spacecraft attitude-control systems in the most fundamental of terms that would be common to all types, i.e., in terms of the resulting angular-acceleration response. Angular-acceleration response was defined as having the following five fundamental characteristics: amplitude, dura-

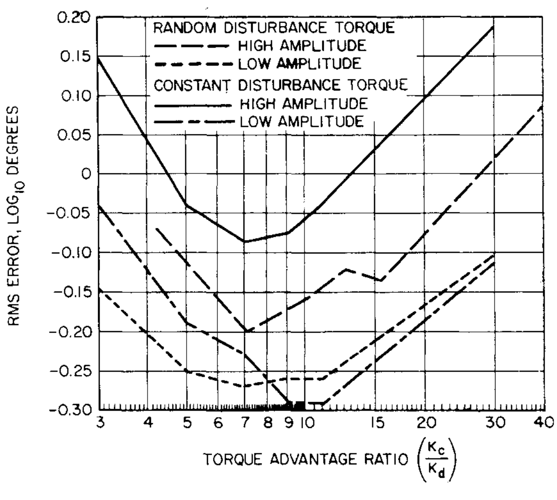

Fig. 1 RMS error as a function of TAR and type of disturbance torque.

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